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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The drive to develop the full strength of the exponential Fourier densities (EFD's) was continued during the time period covered by this report. The EFD's have been successfully applied to the projective 2-space which is the state space of an process. The results were generalized to n-dimensional system. More striking was the generalization to an arbitrary compact Lie group.  A great deal of time was spent to the estimation problem of continuous-time rotational processes. No significant conclusions were reached and the effort			

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20. Abstract

was given up at the end of the reporting period. In its place an investigation of the lack of closure property of EFD's was started. The convolution of an EFD(n) and an EFD(1) was tested for EFD(n)'s on the circle. It was confirmed that for  $n=1$  and 2, the convolution of an EFD(n) and an EFD(1) was uniformly very close to an EFD(n).

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Interim Patent Report  
and  
Interim Scientific Report  
on  
Detection, Estimation, and Control  
on Group Manifolds

No. 3

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FROM: James T. Lo  
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University of Maryland Baltimore County  
Baltimore, Maryland 21228

SUBJECT: Grant No. AFOSR-74-2671B  
Detection, Estimation, and Control  
on Group Manifolds  
Second Interim Patent and Scientific Reports  
covering the period:  
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Interim Report

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Interim Scientific Report

on

Detection, Estimation, and Control

on Group Manifolds

No. 1

Air Force Office of Scientific Research

4325 Jones Road, Dayton, Ohio 45462

1978

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Grant No. AFOSR-78-0155

Detection, Estimation, and Control

on Group Manifolds

Second Interim Report and Scientific Report

covering the period

June 1, 1976 - May 31, 1977

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A. D. BLOSE

Technical Information Officer

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## I. General

These interim reports cover work carried out by faculty and staff members of the Department of Mathematics during the period 1 June 1975 through 31 May 1976 under Grant No. AFOSR-74-2671B.

Progress was mainly contained in the technical papers and reports listed in Section II. The first three items are attached herewith and the last one will be forwarded as soon as completed. The progress will be summarized in Section IV.

During the report period the following people contributed to the project: Associate Professor James T. Lo, Research Assistant Professor Masahiro Nishihama, and Miss Linda R. Eshleman.

## II. Publications

- (1) Exponential Fourier Densities on  $S^2/\pm 1$  and Optimal Estimation for Axial Processes, UMBC Mathematics Research Report No. 77-1, UMBC, Balto., Md., January 1977; submitted for publication in the IEEE Trans. on Information Theory.
- (2) Estimation and Detection on Lie Groups, an invited survey paper to appear in a forthcoming book, Nonlinear Filtering and Estimation Theory--a Status Review, edited by E. B. Stear, Marcel Dekker; also UMBC Mathematics Research Report No. 77-11, UMBC, Balto., Md., October 1977.
- (3) Estimation Problems with Lie Group Structure, an invited paper included in the Proc. of the 1977 IEEE Conference on Decision and Control, New Orleans, Louisiana, December 1977.
- (4) Convolution of Exponential Fourier Densities and Filtering on the Circle, to appear.



### III. Patents

No patents have been obtained and no applications for patents have been filed as a result of the progress being reported on.

### IV. Summary of Progress

The drive to develop the full strength of the exponential Fourier densities (EFD's) was continued during the time period covered by this report. It resulted in the four reports and publications listed in Section II. The EFD's have been successfully applied to the projective 2-space, usually denoted by  $S^2/\pm 1$ , which is the state space of an axial process. The results on  $SO(3)$ ,  $S^2$ , and  $S^2/\pm 1$  can be easily carried over to  $SO(n)$ ,  $S^n$ ,  $S^n/\pm 1$ . However, more striking is the generalization to an arbitrary compact Lie group reported in Publications No. 3 and No. 2.

The research efforts during the year was not all roses. A great deal of time of both the postdoctoral research fellow, Dr. M. Nishihama, and the P.I. was devoted to the estimation problem of continuous-time rotational processes. The problem turned out to be extremely difficult. While many crucial issues were placed in perspective, we were not able to reach any significant conclusion. The effort was finally given up in April, 1977.

The failure on continuous-time systems directed more man power into the research on EFD's. It is known that in spite of all the niceties of the EFD's, they are handicapped by the lack of the closure property under convolution. As a consequence, the signal processes in all the previous results on EFD's do not contain random driving terms. Naturally, how to circumvent this difficiency became the focal point of our effort.

It was suspected by the P.I. as early as 1975 that the convolution of an EFD(n) and an EFD(1) was not too far away from an EFD(n). Long series of Fortran



codes were created to test this conjecture for  $EFD(n)$ 's on the circle. It has been confirmed that for  $n = 1$  and  $2$ , the convolution of an  $EFD(n)$  and an  $EFD(1)$  was uniformly very close to an  $EFD(n)$ . These numerical results and their application to filtering will be documented in Publication 4. The endeavor has involved tremendous amount of program debugging and computation-- more than 2,000 minutes of CPU time on the UNIVAC 1108 of the University of Maryland. This explains in part why the progress on our project has been so sluggish since April 1977.

Before we summarize the results of each of the four papers listed in Section II, it is appropriate to note here that the graduate research assistant, Linda R. Eshleman, supported by the Grant graduated and was granted a Ph.D. degree in Applied Mathematics during the report period.

(1) Exponential Fourier Densities on  $S^2/\pm 1$  and Optimal Estimation for Axial Processes --

In this paper we consider the problem of estimating axes in three-dimensional space. An axis or axial vector is distinguished from a polar vector in that the former is invariant under inversion. Such axes occur in many diverse areas including the following: geophysical fluid dynamics to estimate the vorticity of a flow, paleomagnetism to estimate a magnetic field, crystallography to estimate the optic axis of a crystal, geology to estimate the direction of a normal to the axis of a fold in a layer of rock, and quantum mechanics to estimate the axis of rotation of a rigid body rotation.

Using densities of the form  $\exp f$  where  $f$  is a linear combination of axially symmetric spherical harmonics, estimation problems which arise by examining various possible ways of obtaining a displacement of an axis will be solved in this paper. Although the state space under consideration is homeomorphic to a hemisphere of  $S^2$ , the results for estimation on  $S^2$  cannot be applied

for several important reasons: the displacements defined in that paper may result in a given point being displaced to a non-antipodal point in the opposite hemisphere, the densities on  $S^2$  were not, in general, axially symmetric, and the error criterion used for  $S^2$  is undesirable since it would result in a rejection of the antipode of the optimal estimate.

Using the various displacements and conditional densities obtained in this paper, detection for axial processes would be described by procedures similar to those used for  $S^2$  and  $SO(3)$ .

## (2) Estimation and Detection on Lie Groups --

There are five main sections in this survey chapter to be included in the forthcoming book, Nonlinear Filtering and Estimation Theory--a Status Review, edited by E. B. Stear. They are summarized as follows:

### (2a). PROBABILITY ON THE CIRCLE.

There are many fundamental differences between the estimation and detection problems on the Euclidean spaces and those on the Lie groups. In order for some readers to appreciate them, this section will be addressed to some probabilistic elements on the circle. The probability distribution function and the characteristic function on the circle will first be briefly introduced.

One of the main concerns in this chapter is to study how one uses the knowledge of the probability distribution of a random variable taking values on a Lie group to determine an estimate of the random variable that minimizes a certain error criterion. The conventional least squares technique cannot be used here. Let us take the circle as an example. The square error of the angles  $0^\circ$  and  $359^\circ$  is  $(359^\circ)^2$ , whereas by geometrical intuition they are only  $1^\circ$  apart. In the sequel we will look into this issue on the circle in detail.

The importance of the normal probability densities cannot be over-emphasized for estimation and detection on Euclidian spaces. Unfortunately, there does not exist an analogous density on the circle that possesses all the nice properties of the normal density. In fact, the nice properties of the normal density are almost equally divided between two contenders for normalcy, the folded normal density and the circular normal density. It turns out that while the folded normal density is natural to use for continuous-time estimation, the circular normal density is more suitable for discrete-time estimation. They will both be discussed and compared in this section.

#### (2b). DISCRETE-TIME ESTIMATION ON THE CIRCLE.

Estimation for discrete-time systems on the circle was studied before, using both folded normal densities and Fourier series representations of probability densities. The optimal estimation equations obtained therein are infinite-dimensional and cumbersome. Although some numerical simulation has been done on the suboptimal equations obtained from truncating the higher order terms, it is not clear whether these equations have satisfactory performance in general.

As a matter of fact, the "dimension" of the optimal estimation equations derived from using the folded normal densities increases very rapidly in time. When the Fourier series are used to represent probability densities, the application of Bayes' rule, which involves the multiplication of two a priori densities, has the effect of spreading the dominant Fourier coefficients into the higher order terms. Obviously, this dilemma becomes compounded in multistage estimation problem when a sequence of multiplications of Fourier series takes place.



In this section, we will present an alternative approach. The approach is based on a new class of probability density functions which have the form

$$\exp \sum_{k=0}^n (a_k \cos kx + b_k \sin kx).$$

Such a density will be called an exponential density of order  $n$ , to be denoted by  $\text{EFD}(n)$ .

#### (2c). CONTINUOUS-TIME ESTIMATION ON THE CIRCLE.

A signal process and an observation process, taking values on  $S^1$ , will be formulated in terms of bilinear Ito matrix differential equations. The conditional probability distribution of the signal, given observations over a certain period of time, will be evaluated. Recursive computational schemes for optimal estimation (filtering, smoothing, and prediction), with respect to the error criteria defined in Subsection II.2, will be derived. In fact it will be shown that optimal estimates on  $S^1$  can be obtained recursively by the use of an ordinary vector space estimator together with a nonlinear preprocessor and a nonlinear postprocessor. Multichannel estimation on abelian Lie groups will be examined. Examples illustrating the optimal estimation procedure are given at the end of this section.

#### (2d). DISCRETE-TIME ESTIMATION ON COMPACT LIE GROUPS.

The results of (2b) can be easily generalized to the problems on compact Lie groups by introducing a similar exponential Fourier density (EFD) on the group. This density is obtained by using a sequence of irreducible unitary representations which form a complete orthogonal system on the compact group. It can be shown that a continuous density function on the group can be approximated by such an EFD as closely as we wish in the space of square integrable functions.

As in the circle case, the class of ERD's of a certain finite order on the compact Lie group is closed under the operation of taking conditional distributions as a consequence of the group structure of the group. It will become clear in the sequel that it is exactly this closure property of the EFD's that yields simple estimation schemes which update the sequential conditional densities by recursively revising a finite and fixed number of parameters.

In order to illustrate how the conditional density can be used to calculate the optimal estimate on the group, a rigid body attitude estimation problem is solved as an example. The error criterion, the optimal estimate, and the estimation error with respect to the criterion will be derived for a given probability distribution.

#### (2e). DETECTION FOR CONTINUOUS-TIME SYSTEMS ON LIE GROUPS.

The idea of "rolling without slipping" introduced in Section IV will now be generalized and used to formulate an observation process on an arbitrary matrix Lie group. Briefly speaking, we will inject the differentials of an observation process described by a vector Ito differential equation into a Lie group via the exponential map and then piece them together. The resulting product integral describes our observation process on the Lie group, the injected vector observation process being called its skew form.

The observation process thus constructed on a Lie group will be seen to satisfy a bilinear matrix stochastic differential equation, when its skew form is linear. The observational noise can be viewed as entering multiplicatively.

Given an arbitrary bilinear matrix observation process, we will show that the corresponding skew observation process can be obtained by "reversing" the above injecting procedure. Furthermore, these two procedures will be seen to

induce two "almost sure" bijective mappings between a vector-valued and a matrix-valued function spaces, one being the inverse of the other.

It is well known that the study of a Lie group may be greatly simplified by considering the tangent space (the Lie algebra) of the Lie group at its identity. In fact, the local study of a Lie group is entirely equivalent to the study of the finite dimensional linear algebraic structures of the Lie algebra. In this paper, the above bijective mappings facilitate similar simplification. It enables us to evaluate the likelihood ratio in a finite dimensional linear space--the Lie algebra!

In view of the above construction, the null and the alternative hypotheses that the signal is respectively absent and present in the observation on a Lie group can be written as a pair of bilinear matrix stochastic differential equations. Using the bijective mappings, we may transform these hypotheses on a Lie group into those on the corresponding Lie algebra. There the likelihood ratio can be expressed by the well-known Duncan's formula. Thus the evaluation of the likelihood ratio on a Lie group also hinges on the least-squares estimation.

When the signal is a linear diffusion process, the idea of using the bijective mappings to work in the Lie algebra also leads to a finite dimensional filtering equation for evaluating the least-squares estimate. This equation is indeed an immediate extension of the Kalman-Bucy filter to the case with observation on Lie groups.

### (3) Estimation Problems with Lie Group Structure --

The exponential Fourier densities were used to study estimation on the unit circle, the unit sphere, the three-dimensional rotation group, and the projective two-space in a sequence of recent papers. Many finite-dimensional



optimal estimation schemes were obtained mainly due to the closure property of the exponential Fourier densities of any given finite order under the operation of taking conditional distributions. Another reason for using exponential Fourier densities is that any continuous or bounded-variation probability density on the aforementioned spaces can be very closely approximated by such a density.

It is the purpose of this paper to generalize the previous results to an arbitrary compact Lie group and thereby to illustrate that it is the structure of a compact Lie group that accounts for the usefulness of the exponential Fourier densities.

As it is expected that most readers of this paper will be engineers, some definitions and preliminary results will be briefly summarized below to facilitate our presentation in the sequel.

(4) Convolution of Exponential Fourier Densities and Filtering on the Circle --

While the  $EFD(n)$ 's are closed under conditioning, they are not closed under convolution. This deficiency has prevented us from including random driving terms in the signal processes in our results using  $EFD(n)$ 's. In this paper, we will present a striking property of the  $EFD(n)$ 's; namely, they are almost closed under convolution.

The maximum informational distance, in the sense of Kullback, between the convolution of two  $EFD(1)$ 's and its best fit by an  $EFD(1)$  is numerically calculated and is 0.00539412984011850. Such a small number indicates that the convolution of any two  $EFD(1)$ 's is virtually an  $EFD(1)$ . Hence it is not surprising that replacing the convolution of two a priori  $EFD(1)$ 's with its best fit  $EFD(1)$  yields a near optimal estimate, which is almost indistinguishable from the optimal one. All these numerical results are reported in the paper.

The case of  $EFD(2)$ 's will also be thoroughly studied in the paper. More details will be submitted to AFOSR as soon as available.